The Gamma function and its functional inverse

David Jeffrey
Department of Applied Mathematics
School of Mathematical and Statistical Sciences
The University of Western Ontario, Canada

A recent review [1] of the factorial or Γ function noticed some interesting facts. The asymptotic approximation known as Stirling's formula was really due to de Moivre, and Stirling's own formula is actually more accurate. Moreover, derivations of Stirling's formula are very popular amongst contributors to the American Math Monthly: about 30 variations on the derivation have been published. For many authors, the derivation of the formula is more important than its utility. In fact I shall show it is remarkably accurate even in the complex domain.

Another observation was that only a few papers have addressed the question of the functional inverse of Γ. One of the earliest applications of inverse Γ was made by Gaston Gonnet in 1981 [2] (in the same paper that defined W). The inverse is a new challenge for my approach to understanding multivalued inverse functions in the complex plane [3]. I shall describe work on evaluation on the real line and the description of branches in the complex plane.

References

[1] J. M. BORWEIN AND R. M. CORLESS, Gamma and factorial in the Monthly, *American Math. Monthly*, **125**, 400–424, (2018).

[2] G. H. GONNET, Expected length of the longest probe sequence in hash code searching, J. of ACM, **28**, 289–304 (1981)

[3] D. J. JEFFREY Twenty years of Lambert W. RISC seminar 2016.