WEIRD GRÖBNER BASES: AN APPLICATION OF AC CONGRUENCE CLOSURE

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A Gröbner basis algorithm for polynomial ideals over the integers is presented as an instance of a congruence closure algorithm of ground equations with two associative-commutative (AC) operations with $+$ and $*$, additionally satisfying the properties of a commutative ring with a unit. This view of a Gröbner basis algorithm is in sharp contrast to viewing it as an instance of a more general Knuth-Bendix completion procedure, when generalized to AC symbols and/or richer equational theories. The proposed view is more flexible in the use of termination orderings on polynomials, leading to generation of "weird" Gröbner bases by introducing new symbols to stand for power products, and resulting in more flexible canonical forms in quotient ring defined by a polynomial ideal. This view easily extends when coefficients are from a finite field defined by $\mathbb{Z}_p$, p is prime, and a ring $\mathbb{Z}_q$ with zero divisor, where q is not prime.